

3 Complementarity

 Name

Learning objectives

- Define and formulate variational inequality problems
- Define and formulate complementarity problems
- Explain the equivalence between variational inequalities and complementarity problems
- Explain in which cases an optimization problem can be formulated as a complementarity problem and viceversa
- Solve complementarity problems using PATH solver under GAMS
- Explain some of the algorithms used to solve linear complementarity problems

Bibliography

- Basic reading:
 - Facchinei, F. and Jong-Shi P. Finite-dimensional variational inequalities and complementarity problems. Vol. 1. New York: Springer, 2003 (chapter 1).
 - Gabriel, S., Conejo. A., Fuller, D., Hobbs, B., and Ruiz, C. Complementarity modeling in energy markets. Springer, 2012 (chapters 5 and 8).
- Further reading:
 - Cottle, R. W., Jong-Shi Pang, and Richard E. Stone. The linear complementarity problem. Vol. 60. Siam, 2009.

Variational inequalities

 Discuss what is obtained if the set K of the variational inequality is the whole vectorial space $\mathbb{R}^n$

 Discuss the similarity between a variational inequality and the optimality conditions of a convex optimization problem



Complementarity problem



Proof of relation between complementarity problem and variational inequalities

Nonlinear complementarity problem

Mixed complementarity problem

Linear complementarity problem

 Discuss whether the optimality conditions of an unconstrained problem can be formulated as a nonlinear/mixed/linear complementarity problem

 Discuss whether the optimality conditions of an inequality constrained problem can be formulated as a nonlinear/mixed/linear complementarity problem

 Discuss whether the optimality conditions of an inequality and equality constrained problem can be formulated as a nonlinear/mixed/linear complementarity problem

 Discuss whether a nonlinear complementarity problem can always be formulated as an optimization problem

Example 3-2: Complementarity formulation for generation scheduling

Formulate the KKT conditions of Example 1-3 as a mixed complementarity problem.

Solving Example 3-2 with GAMS (Example3-2.gms)

Solve the complementarity formulation corresponding to Example 1-3 using the same data set and the PATH solver. Compare the results with those previously obtained.

p (€/MWh)	10	20	30	40	50
x (MWh)					

Example 3-3: Complementarity formulation for economic dispatch problem

Formulate the KKT conditions of Example 1-4 as a mixed complementarity problem.

Solving Example 3-3 with GAMS (Example3-3.gms)

Solve the complementarity formulation corresponding to Example 1-4 using the same data set and the PATH solver. Compare the results with those previously obtained.

d (MWh)	100	200	300	400	500
x_{g1} (MWh)					
x_{g2} (MWh)					
x_{g3} (MWh)					



Discuss possible strategies to solve a linear complementarity problem when the matrix M is positive semidefinite.



Example 3-4: Solving complementarity model for economic dispatch problem

Formulate the complementarity model of Example 3-2 as a linear complementarity problem. Find M and q . Is M positive semidefinite? *Tip: eliminate variable x .*



Formulate an optimization equivalent to the linear complementarity problem of Example 3-7 and solve it using GAMS (Example3-7.gms).

Example 3-5: Solving linear program using simplex

Solve the following linear optimization problem using the simplex method.

Maximize $_{x_1, x_2}$

s.t.

$$x_1 + x_2 \leq 3$$

$$x_1 + 2x_2 \leq 4$$

$$x_1 \geq 0, x_2 \geq 0$$

📌 Example 3-6: Solving linear complementarity problem using Lemke's method

Solve the following linear complementarity problem using Lemke's method.

$$0 \leq z_1 \perp z_1 + z_2 \geq 0$$

$$0 \leq z_2 \perp 2z_1 + z_2 - 2 \geq 0$$

✚ Example 3-7: Propose a different example of a linear complementarity problem and solve it using Lemke's method